# Proof Theory for Group-Like Structures

George Metcalfe

Mathematical Institute, University of Bern, Switzerland
george.metcalfe@math.unibe.ch

A central goal of structural proof theory is the development of analytic proof systems for logics and classes of structures that can be used to investigate their algorithmic and model-theoretic properties, notably, decidability and complexity bounds, (uniform) interpolation and amalgamation, and admissible rules and generation by subclasses. Although this endeavour has been successful for broad families of non-classical logics, it hits a roadblock when confronted with some of the most studied structures in mathematics, in particular, structures related in some way to groups. Not only is this an unfortunate limitation on the scope of proof-theoretic methods for tackling problems in algebra, these structures also serve as semantics for a wide range of substructural and many-valued logics.

In this talk, I will explore some recent attempts to address these limitations. First, I will explain how proof systems for classes of ordered groups introduced in [5, 3] relate to total orders on free groups [2, 1] and can be used to establish various decidability, complexity, and generation results. In the second part of the talk, I will consider how these results for ordered groups can be extended, via a Glivenko-style theorem, to classes of residuated lattices with close connections to BCI-algebras, Dubreil-Jacotin semigroups, and Casari's comparative logic [4]. Finally, I will describe some of the many open problems for this topic.

## References

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