ARE STRICT CONDITIONALS BUNCHED?

TADEUSZ LITAK

Weiss [Wei19] studies the minimal logic ICK of the intuitionistic propositional calculus (IPC) expanded with normal conditional implication $\square\!\!\rightarrow$. Two such conditionals seem of particular importance: *constructive strict implication* $\prec\!\!3$ and *bunched implication* or *magic wand* $-\!\!*$ of separation logic.

Classically, $\prec\!\!3$ provided the original setting for what is known today as *modal logic* [LL32]. It thus inherits all interpretations of classical normal modal $\square$. Moreover, in the constructive setting new interpretations and applications emerge, ranging from metatheory of arithmetic to functional programming [LV18]. Even its relational interpretation intuitionistically does not collapse to standard Kripkean $\square$ [IDJZ05].

On the other hand, $-\!\!*$ is just substructural implication of Lambek's FL_e combined (fibred, dovetailed) with IPC and its $\rightarrow$. The resulting logic of *bunched implications* BI [OP99, POY04] has found major computer science applications in reasoning about shared mutable data structures. In the recent non-commutative generalization of BI to GBI [GJ17, JL18] one can interpret $\square\!\!\rightarrow$ as either the left residual or the right residual of the substructural fusion connective.

In this talk, I am going to compare and contrast the propositional laws and axioms for these two interpretations. In particular, I am going to show that a reasonable common subsystem is a proper extension of the minimal one proposed by Weiss [Wei19]. If time allows, I am also going to discuss some semantics, including generalizations of neighbourhood frames and Veltman semantics.

References

[GJ17]   Nikolaos Galatos and Peter Jipsen. Distributive residuated frames and generalized bunched implication algebras. *Algebr. Univ.*, 78:303–336, 2017.
[IDJZ05] Rosalie Iemhoff, Dick De Jongh, and Chunlai Zhou. Properties of intuitionistic provability and preservativity logics. *Log. J. IGPL*, 13(6):615–636, 2005.
[JL18]   Peter Jipsen and Tadeusz Litak. An algebraic glimpse at bunched implications and separation logic. In *Hiroakira Ono on Residuated Lattices and Substructural Logics*, Outstanding Contributions to Logic. Springer, 2018. To appear, URL: `https://arxiv.org/abs/1709.07063`.
[LL32]   Clarence I. Lewis and Cooper H. Langford. *Symbolic Logic*. Dover, 1932.
[LV18]   Tadeusz Litak and Albert Visser. Lewis meets Brouwer: constructive strict implication. *Indag. Math.*, 29:36–90, 2018. A special issue "L.E.J. Brouwer, fifty years later", URL: `https://arxiv.org/abs/1708.02143`.
[OP99]   Peter W. O'Hearn and David J. Pym. The logic of bunched implications. *B. Symb. Log.*, 5(2):215–244, 1999.
[POY04] David J. Pym, Peter W. O'Hearn, and Hongseok Yang. Possible worlds and resources: the semantics of BI. *Theoret. Comput. Sci.*, 315(1):257–305, 2004.
[Wei19] Yale Weiss. Basic intuitionistic conditional logic. *J. Philos. Logic*, 48(3):447–469, 2019.

INFORMATIK 8, FAU ERLANGEN-NÜRNBERG, MARTENSSTRASSE 3, 91058 ERLANGEN, GERMANY
Email address: `tadeusz.litak@fau.de`