Logics of finite depth: problems and results

Valentin Shehtman *

We consider modal and intermediate propositional logics of finite depth and their first-order extensions. The depth of a logic can be defined either in syntactic or semantical terms, and there are several variations of this definition. We consider two basic cases: intransitive depth depending on the formulas $\Box^n \bot$ and transitive depth depending on special modal axioms $bd_n$ or intuitionistic $ibd_n$.

The propositional logics of finite depth are locally finite, and the main technical problem is finding the size of their finite Lindenbaum algebras. For the first-order versions different problems arise:

• First-order logics of finite depth are often Kripke incomplete, so other semantics are needed.

• Gödel — Tarski translation from intermediate to modal logics and the translation from extensions of Grz to extensions of GL may be not faithful.

• One-variable fragments of first-order logics may be not simply axiomatizable, and their decidability is unknown.

Still some positive results can be obtained:

• Local finiteness holds for the semi-commutative join of $K + \Box^n \bot$ with $S5$. This implies decidability of one-variable predicate intransitive logics of finite depth. In particular, the finite depth versions of Artemov – Dzhaparidze logic are well-behaved.

• Similar results hold for extensions of Grz of finite depth, and there is a connection between predicate finite depth extensions of Grz and GL.

• The semi-commutative join of a locally finite modal logic $L$ with $S5$ has the finite model property, and in some cases this is exactly the one-variable first-order version of $L$.

• In some cases there is a faithful translation from predicate intermediate logics of finite depth to predicate extensions of Grz.

These results are proved by applying different methods: bismulation games; filtrations; Kripke bundle, functor, and simplicial semantics for predicate logics. This work is supported by Russian Science Foundation (project No. 16-11-10252).

*Institute for Information Transmission Problems, RAS; Steklov Mathematical Institute; Moscow State University; National Research University Higher School of Economics, Moscow, Russia. shehtman@netscape.net