Logics of finite depth: problems and results

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We consider modal and intermediate propositional logics of finite depth and their first-order extensions. The depth of a logic can be defined either in syntactic or semantical terms, and there are several variations of this definition. We consider two basic cases: intransitive depth depending on the formulas $\Box^n\bot$ and transitive depth depending on special modal axioms bd_n or intuitionistic ibd_n.

The propositional logics of finite depth are locally finite, and the main technical problem is finding the size of their finite Lindenbaum algebras. For the first-order versions different problems arise:

- First-order logics of finite depth are often Kripke incomplete, so other semantics are needed.
- Gödel — Tarski translation from intermediate to modal logics and the translation from extensions of **Grz** to extensions of **GL** may be not faithful.
- One-variable fragments of first-order logics may be not simply axiomatizable, and their decidability is unknown.

Still some positive results can be obtained:

- Local finiteness holds for the semi-commutative join of $\mathbf{K} + \Box^n\bot$ with **S5**. This implies decidability of one-variable predicate intransitive logics of finite depth. In particular, the finite depth versions of Artemov – Dzhaparidze logic are well-behaved.
- Similar results hold for extensions of **Grz** of finite depth, and there is a connection between predicate finite depth extensions of **Grz** and **GL**.
- The semi-commutative join of a locally finite modal logic L with **S5** has the finite model property, and in some cases this is exactly the one-variable first-order version of L.
- In some cases there is a faithful translation from predicate intermediate logics of finite depth to predicate extensions of **Grz**.

These results are proved by applying different methods: bismulation games; filtrations; Kripke bundle, functor, and simplicial semantics for predicate logics.

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