

# Direct interpolation for modal $\mu$ -calculus

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Modal logics are known to widely enjoy interpolation and modal  $\mu$ -calculus does so in a very strong sense: given a formula  $A$  and a finite set of propositions and modality operators  $L$ , there exists a formula  $B$  (an interpolant) in the language common to  $A$  and  $L$  such that  $A \rightarrow B$  is valid and for *every* formula  $C$  from  $L$ ,  $A \rightarrow C$  is valid if and only if  $B \rightarrow C$  is valid. This property, called *uniform interpolation*, easily implies Craig interpolation and was established for modal  $\mu$ -calculus by D’Agostino and Hollenberg [2] utilising (disjunctive) modal automata to show that bisimulation quantifiers are definable in modal  $\mu$ -calculus and can readily be used to obtain interpolants.

Aside from the method of propositional quantification, there are syntactical approaches to interpolation apt for non-classical and modal logics. Following the proof-theoretic tradition, in this talk I will show how to directly extract interpolants for modal  $\mu$ -calculus formulæ from the ‘cyclic’ proofs introduced in [1]. The interpolants obtained in this way are structurally identical to the proofs witnessing the interpolated implication and from this observation one can infer results on the logical form of the interpolant.

## References

- [1] Bahareh Afshari and Graham E. Leigh. Cut-free completeness for modal  $\mu$ -calculus. In *Proceeding of Thirty-Second Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*, Lecture Notes in Computer Science. Springer, 2017.
- [2] Giovanna D’Agostino and Marco Hollenberg. Logical questions concerning the  $\mu$ -calculus. *The Journal of Symbolic Logic*, 65(1):310–332, 2000.